Embedded Real-Time Systems

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Based on slides kindly provided by Edward A. Lee & Sanjit Seshia,

Lecture 6a: Synchronous/ Reactive Models

The 5-Minute Review Session

1. What is the *actor model for FSMs*? What is the motivation for it?
2. What does *synchronous composition* mean for FSMs? What are the alternatives?
3. How does a *hierarchical state machine* react?
4. What is a *reset transition*? What is its alternative? How do they compare wrt state space?
5. What is a *preemptive transition*?

Concurrent Composition: Alternatives to Threads

Threads yield incomprehensible behaviors.

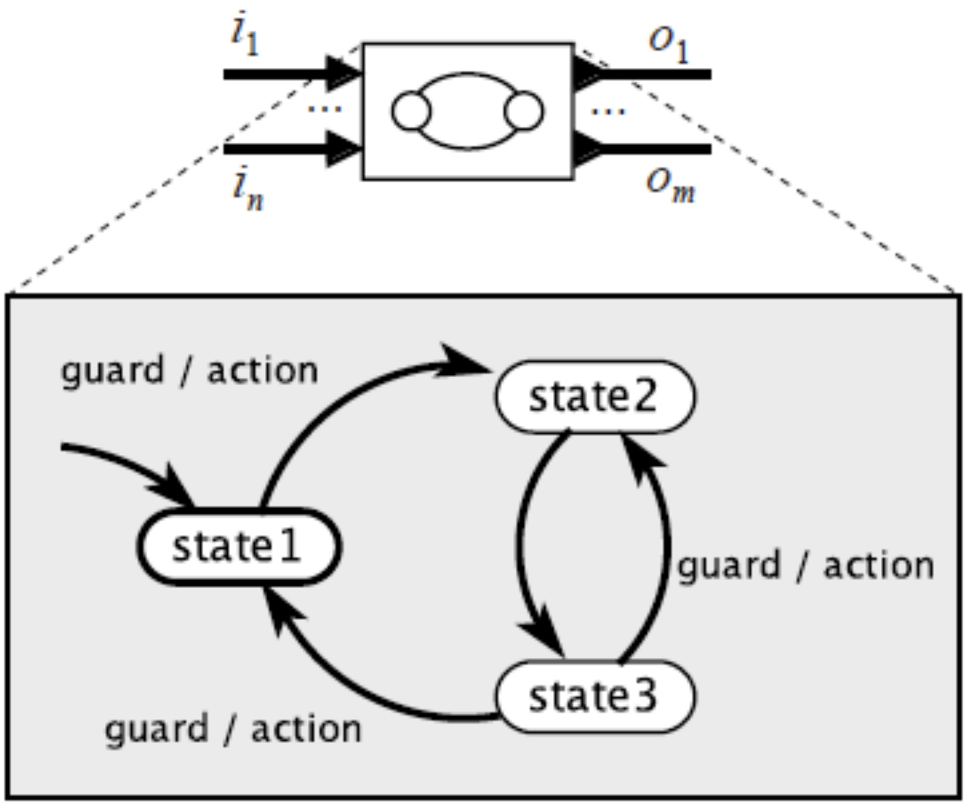
Composition of State Machines:

- Side-by-side composition
- Cascade composition
- Feedback composition

We will begin with synchronous composition, an abstraction that has been very effectively used in hardware design and is gaining popularity in software design.

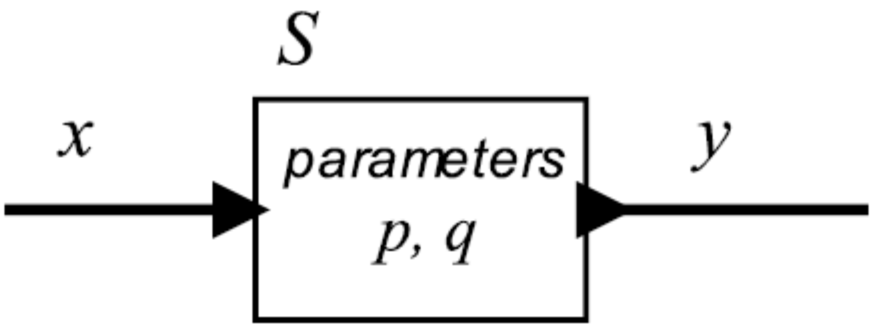
Recall: Actor Model for State Machines

Expose inputs and outputs, enabling composition:



Recall: Actor Model of Continuous-Time Systems

A *system* is a function that accepts an input *signal* and yields an output signal.



The domain (*Definitionsmenge*) and range (*Zielmenge*) of the system function are sets of signals, which themselves are functions.

$$x : \mathbb{R} \to \mathbb{R}, \quad y : \mathbb{R} \to \mathbb{R}$$

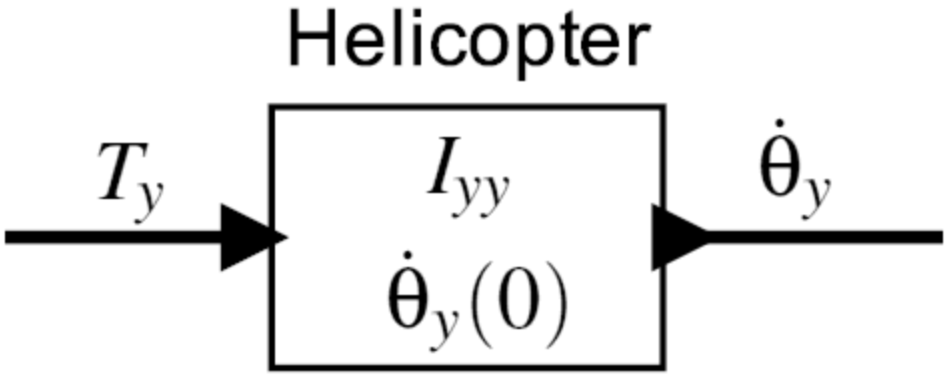
$$S : X \to Y$$

$$X = Y = (\mathbb{R} \to \mathbb{R})$$

Parameters may affect the definition of the function *S*.

Example: Actor model of the helicopter

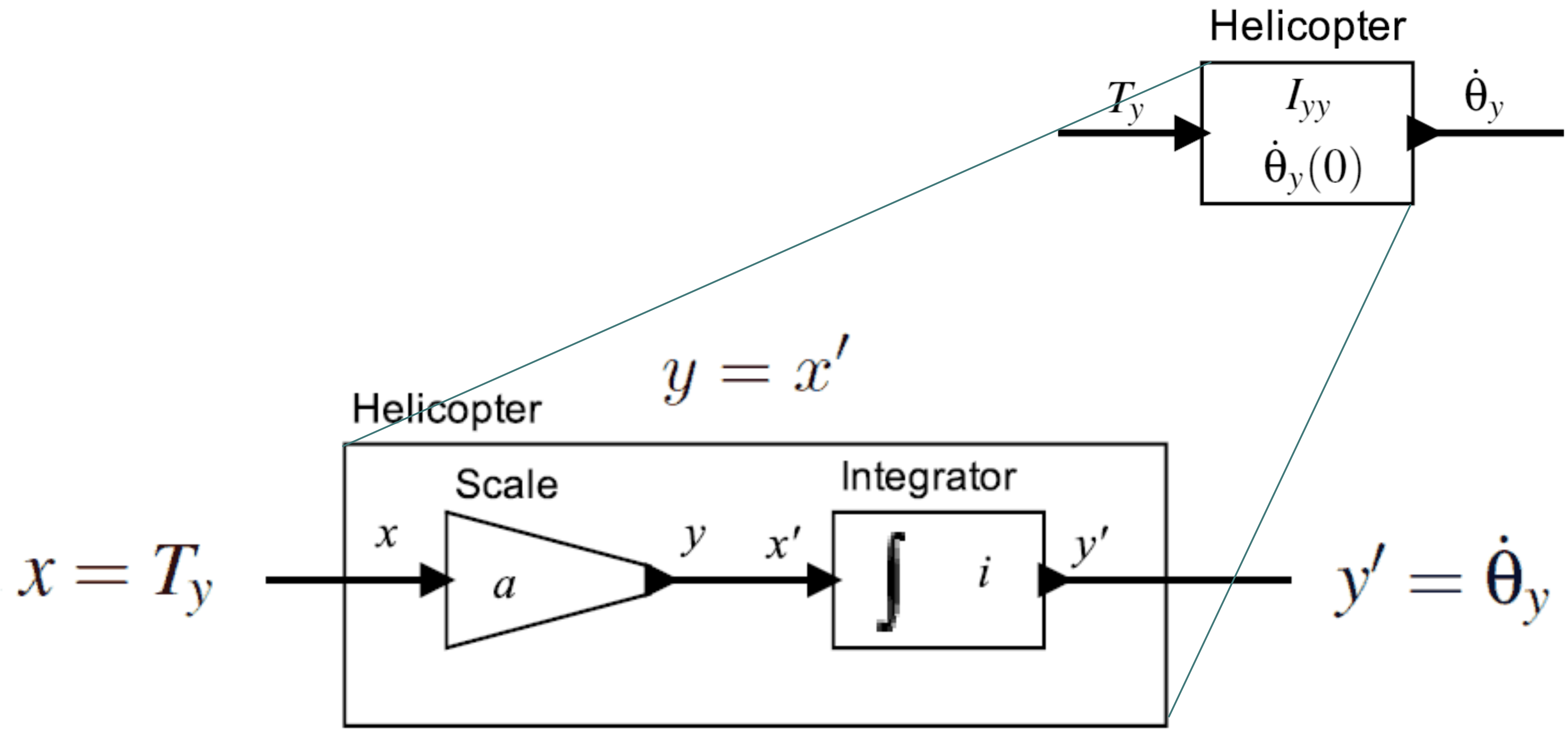
Input is the net torque of the tail rotor and the top rotor. Output is the angular velocity around the *y* axis.



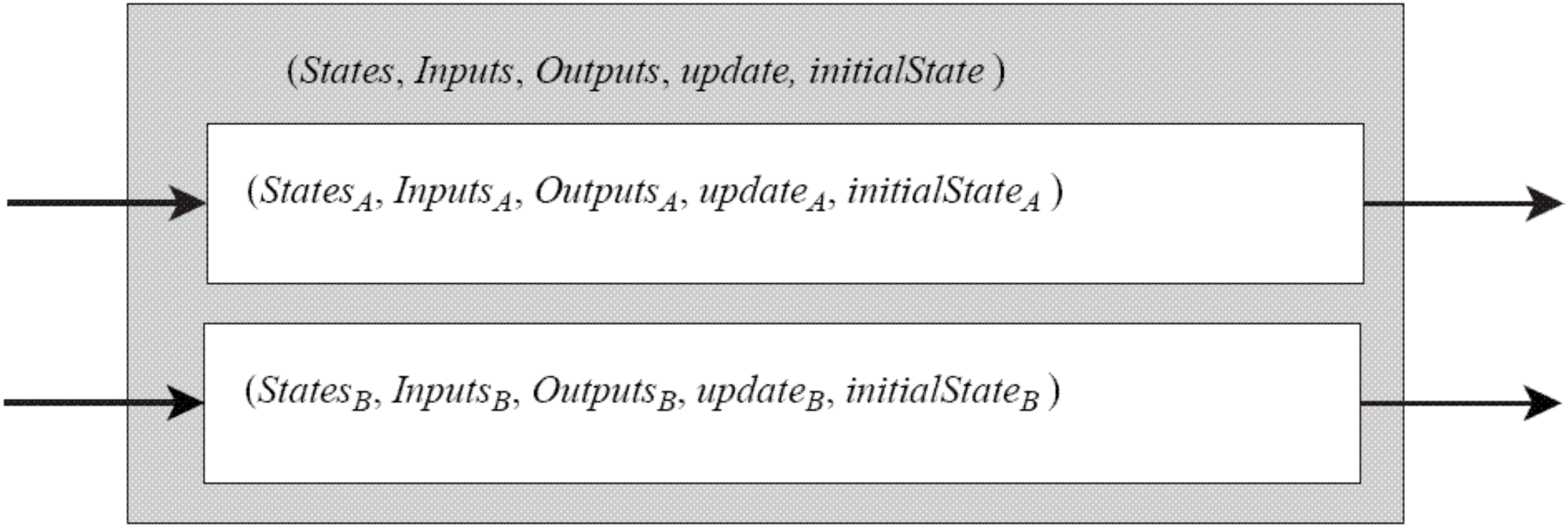
Parameters of the model are shown in the box. The input and output relation is given by the equation to the right.

$$\dot{\theta}_y(t) = \dot{\theta}_y(0) + \frac{1}{I_{yy}} \int_0^t T_y(\tau) d\tau$$

Recall: Composition of actor models

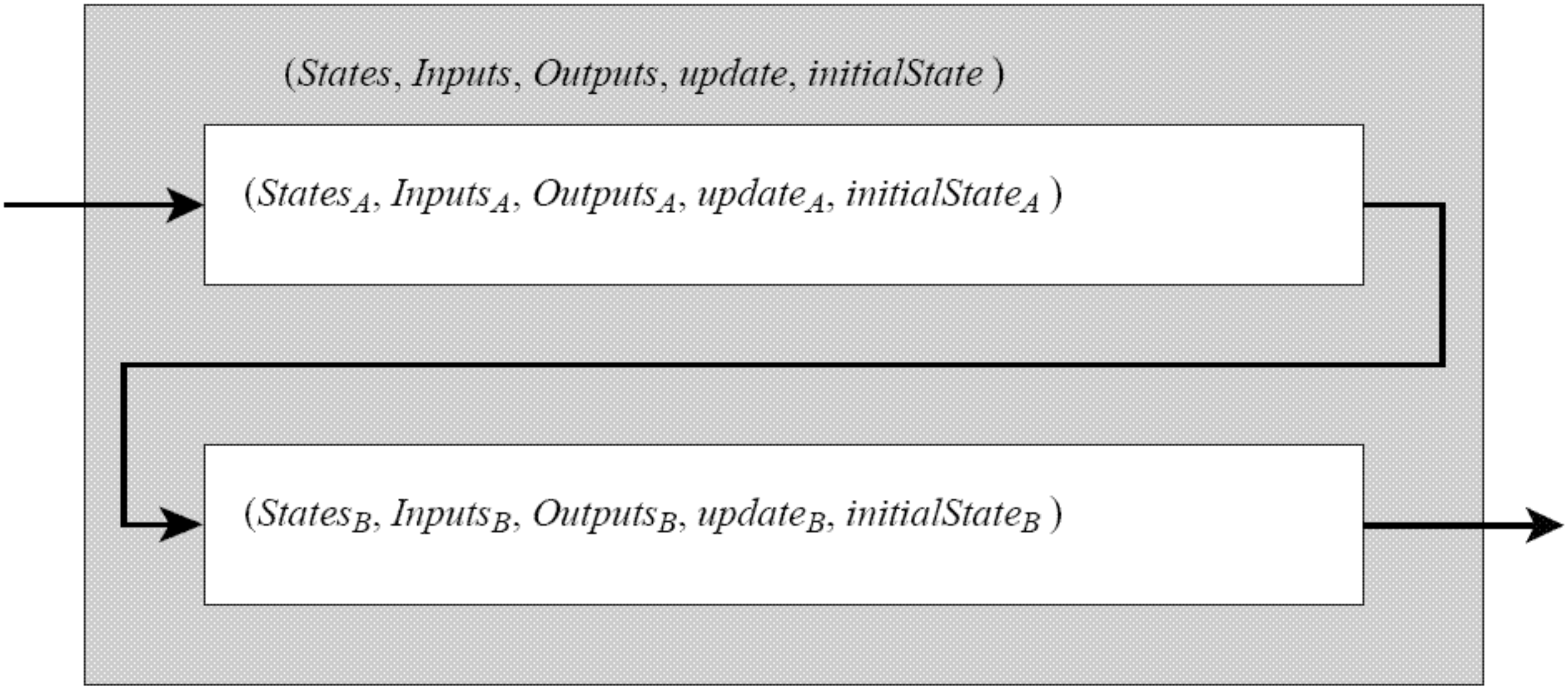


Side-by-Side Composition



Synchronous composition: the machines react simultaneously and instantaneously.

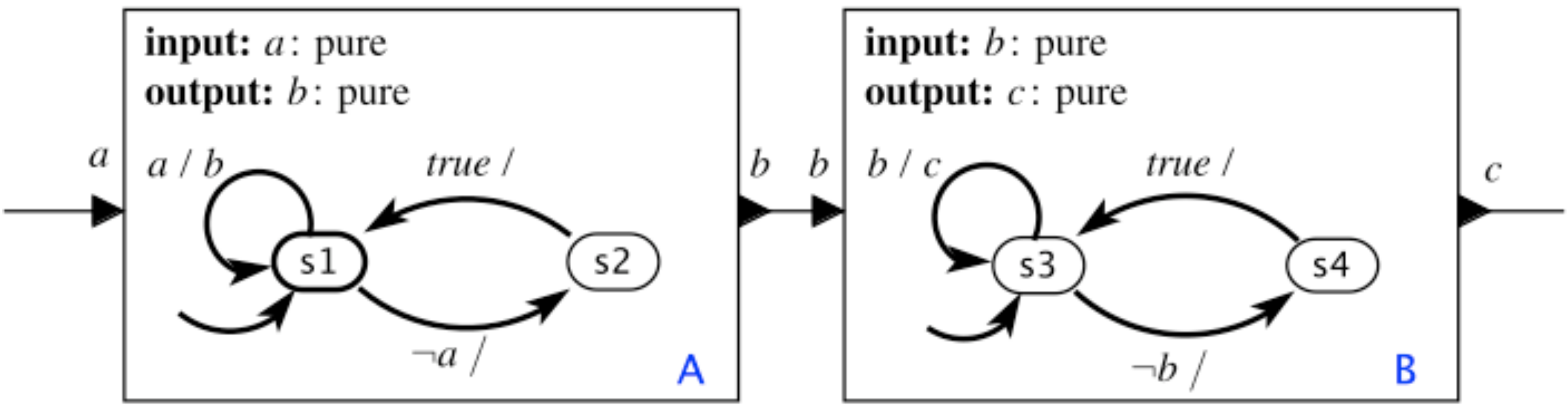
Cascade Composition



Synchronous composition: the machines react simultaneously and instantaneously, despite the apparent causal relationship!

Synchronous Composition:
Reactions are *Simultaneous* and *Instantaneous*

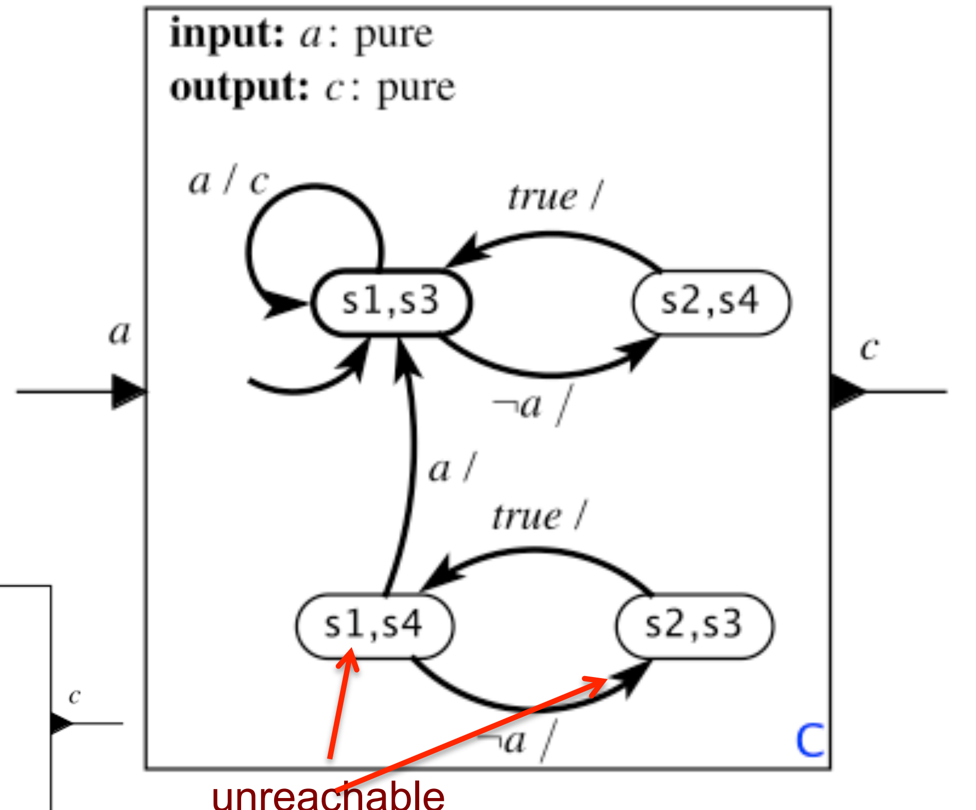
Consider a cascade composition as follows:

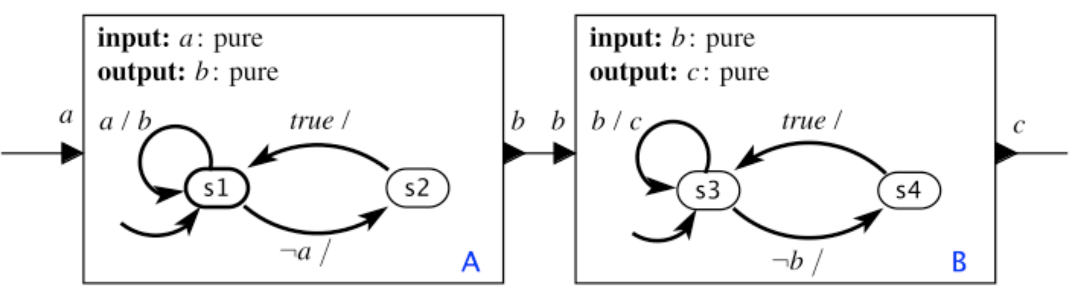


Synchronous Composition:
Reactions are *Simultaneous* and *Instantaneous*

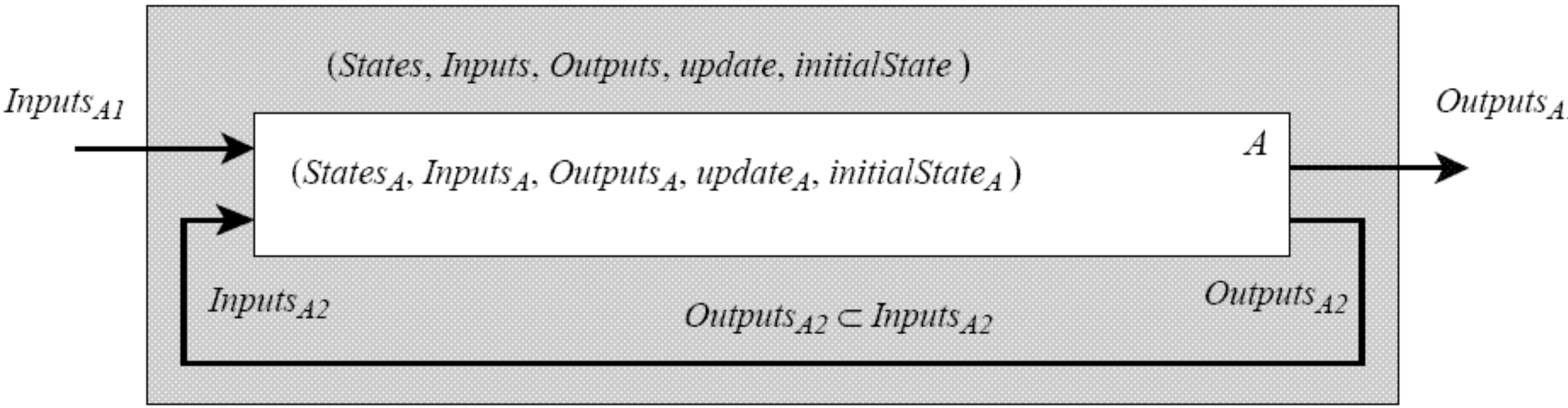
In this model, you must not think of machine A as reacting before machine B. If it did, the unreachable states would not be unreachable.

$$S_C = S_A \times S_B$$



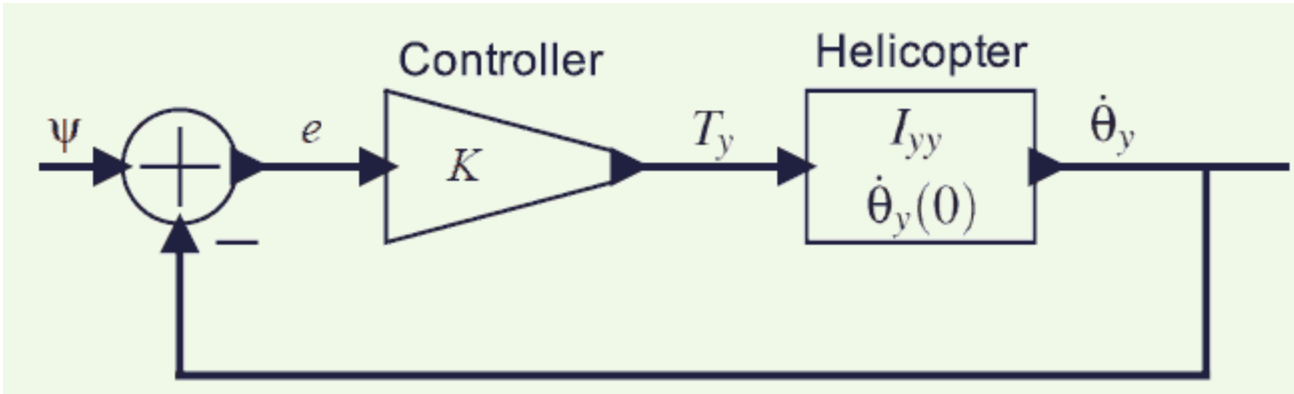


Feedback Composition



Turns out everything can be viewed as feedback composition…

Example: Feedback Composition

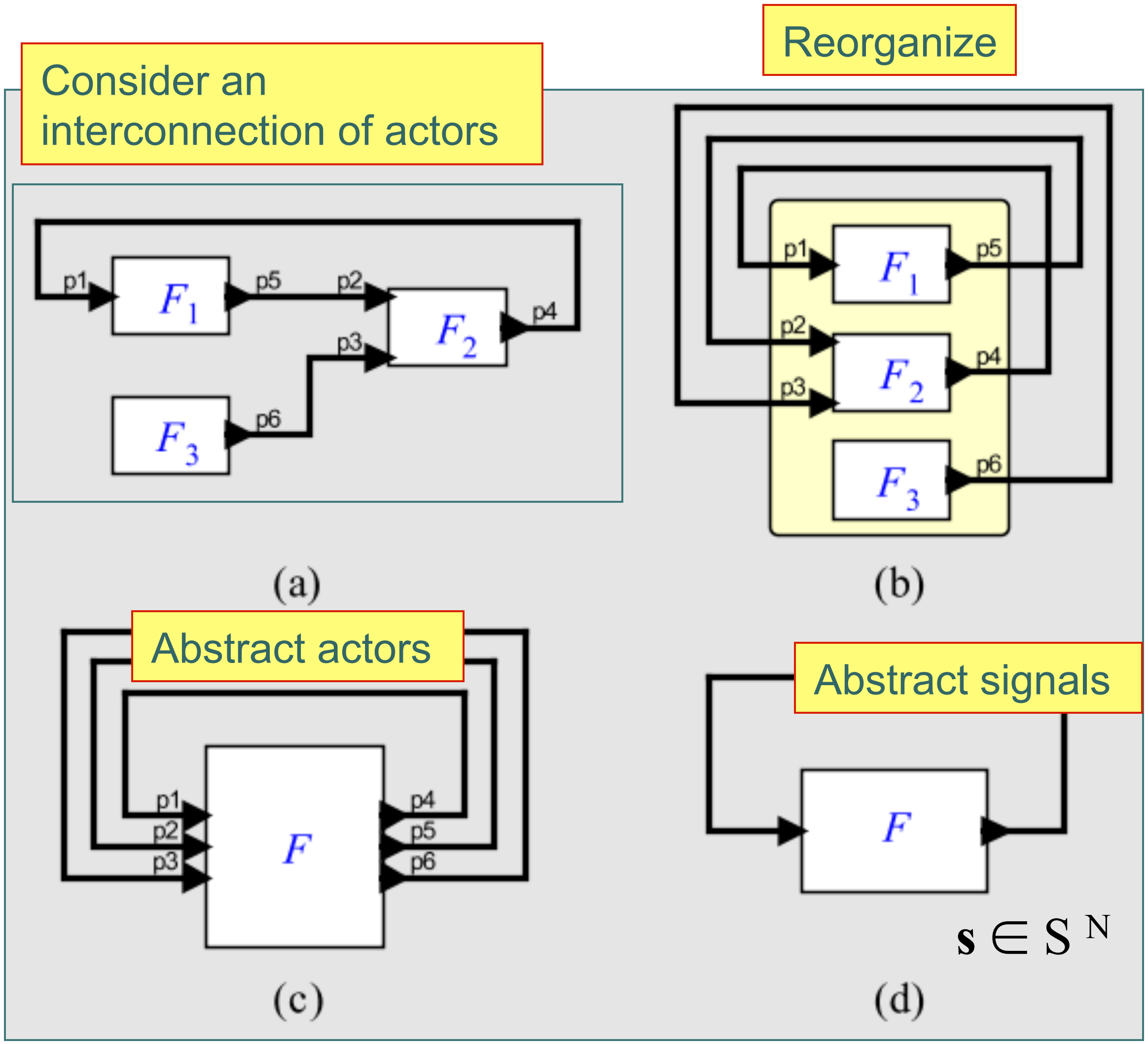


$$\dot{\theta}_y(t) = \dot{\theta}_y(0) + \frac{1}{I_{yy}} \int_0^t T_y(\tau) d\tau$$

$$= \dot{\theta}_y(0) + \frac{K}{I_{yy}} \int_0^t (\psi(\tau) - \dot{\theta}_y(\tau)) d\tau$$

Angular velocity appears on both sides. The semantics (meaning) of the model is the solution to this equation.

Observation: Any Composition is a Feedback Composition



We seek an $\mathbf{s} \in S^N$ that satisfies $F(\mathbf{s}) = \mathbf{s}$.

Such an **s** is called a ***fixed point***.

We would like the fixed point to exist and be unique. And we would like a constructive procedure to find it.

It is the *behavior* (*semantics*) of the system.

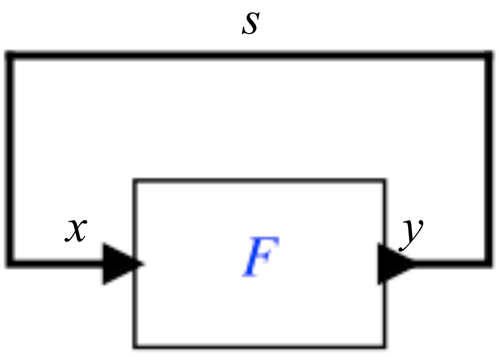
Data Types

As with any connection, we require compatible data types:

$$V_y \subseteq V_x$$

Then the signal on the feedback loop is a function

$$s\colon \mathbb{N} \to V_y \cup \{absent\}$$



Then we seek s such that

$$F(s) = s$$

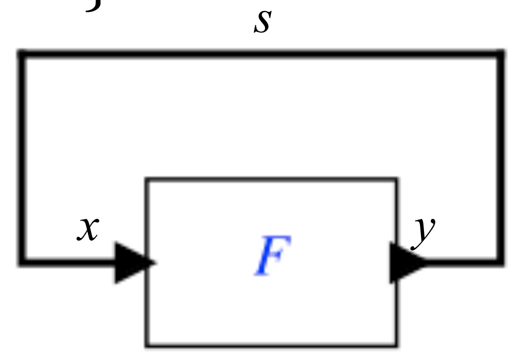
where F is the actor function, which for determinate systems has form

$$F\colon (\mathbb{N} \to V_x \cup \{absent\}) \to (\mathbb{N} \to V_y \cup \{absent\})$$

Firing Functions

With synchronous composition of determinate state machines, we can break this down by reaction. At the *n*-th reaction, there is a (state-dependent) function

$$f(n) : V_x \cup \{absent\} \to V_y \cup \{absent\}$$



such that

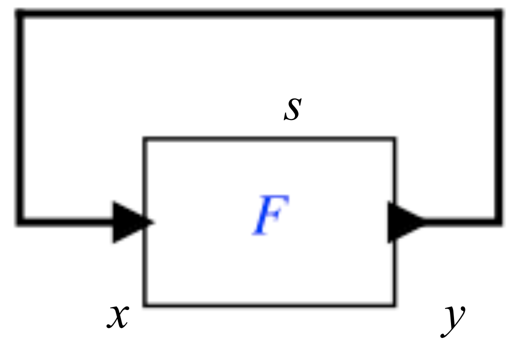
$$s(n) = (f(n))(s(n))$$

This too is a fixed point.

Well-Formed Feedback

At the n-th reaction, we seek $s(n) \in V_y \cup \{absent\}$ such that

$$s(n) = (f(n))(s(n))$$



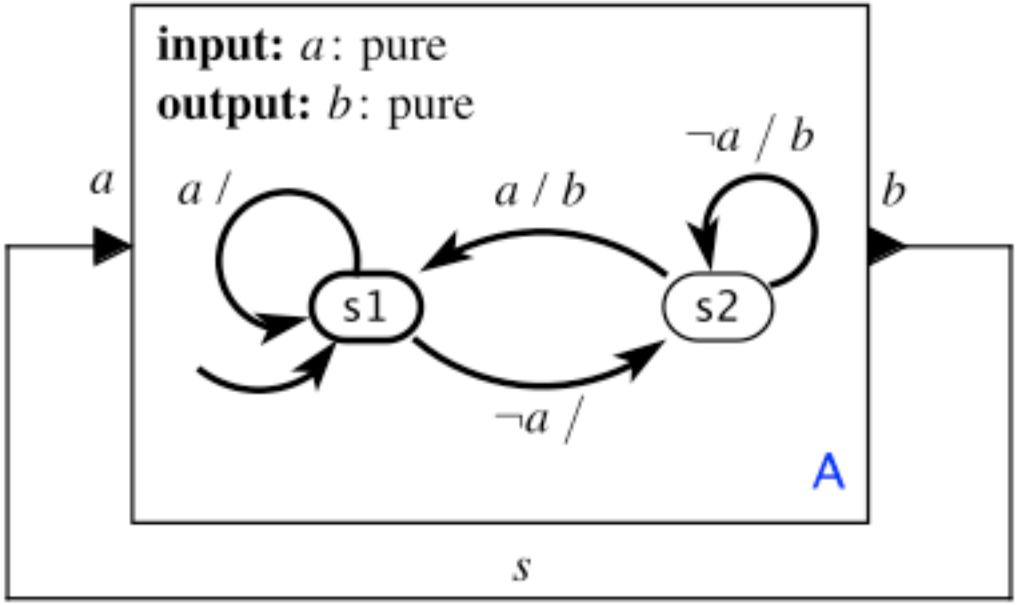
There are two potential problems:

1. It does not exist.
2. It is not unique.

In either case, we call the system **ill formed**. Otherwise, it is **well formed**.

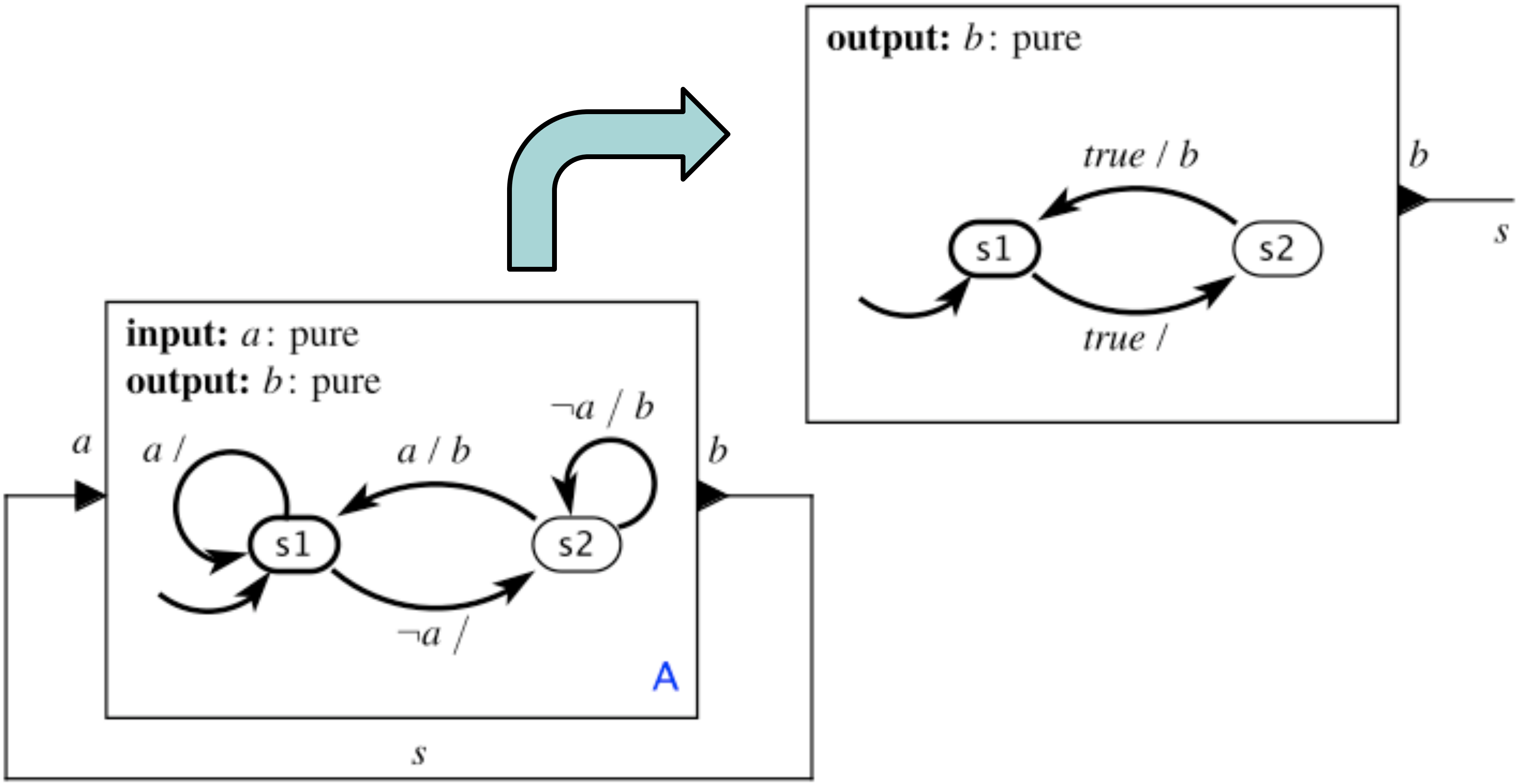
Note that if a state is not reachable, then it is irrelevant to determining whether the machine is well formed.

Well-Formed Example

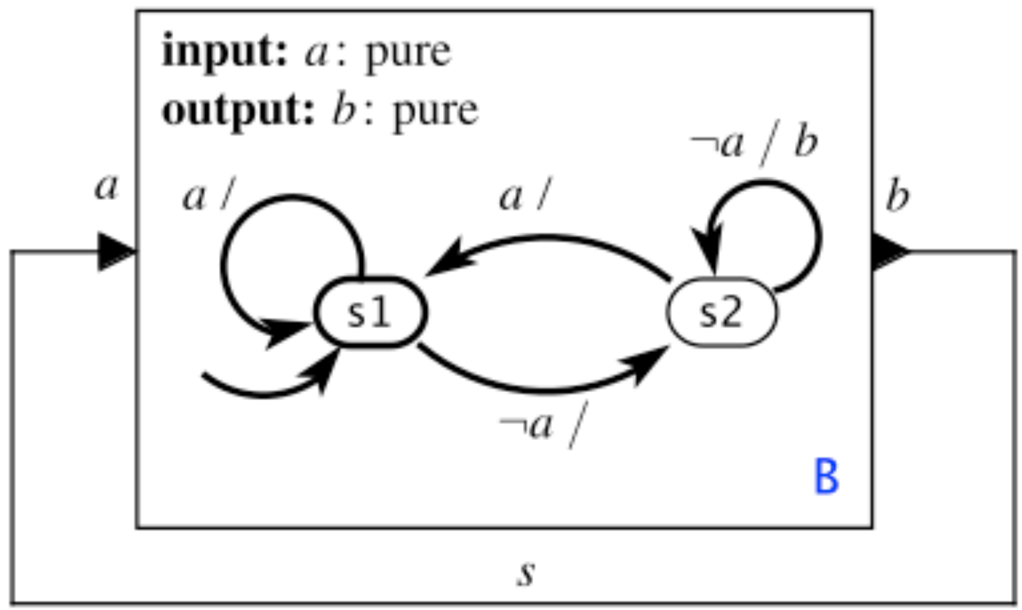


In state s1, we get the unique $s(n) = absent$.
In state s2, we get the unique $s(n) = present$.
Therefore, s alternates between *absent* and *present*.

Composite Machine

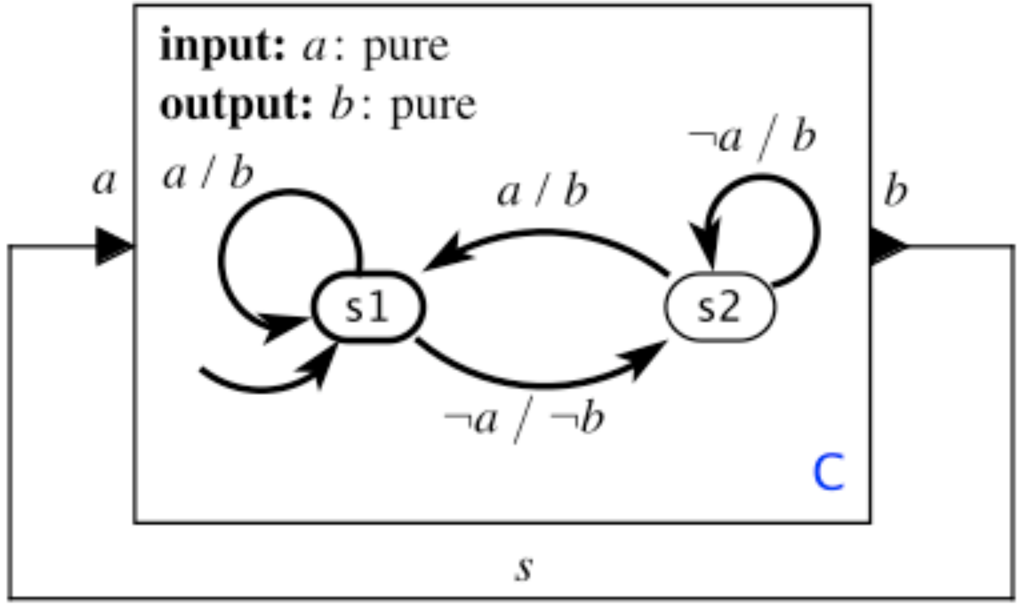


Ill-Formed Example 1 (Existence)



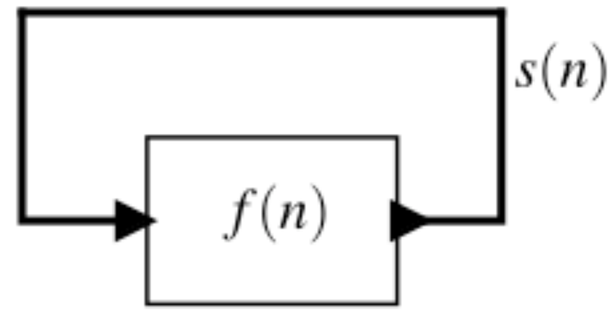
In state s1, we get the unique $s(n) = absent$.
In state s2, there is no fixed point.
Since state s2 is reachable, this composition is ill formed.

Ill-Formed Example 2 (Uniqueness)



In s1, both $s(n) = absent$ and $s(n) = present$ are fixed points.
In state s2, we get the unique $s(n) = present$.
Since state s1 is reachable, this composition is ill formed.

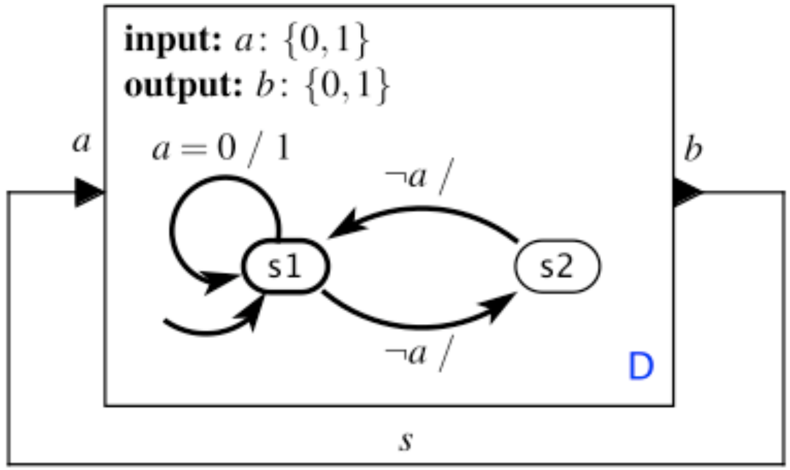
Constructive Semantics: Single Signal



1. Start with $s(n)$ *unknown*.
2. Determine as much as you can about $(f(n))(s(n))$.
3. If $s(n)$ becomes known (whether it is present, and if it is not pure, what its value is), then we have a unique fixed point.

A state machine for which this procedure works is said to be **constructive**.

Non-Constructive Well-Formed State Machine

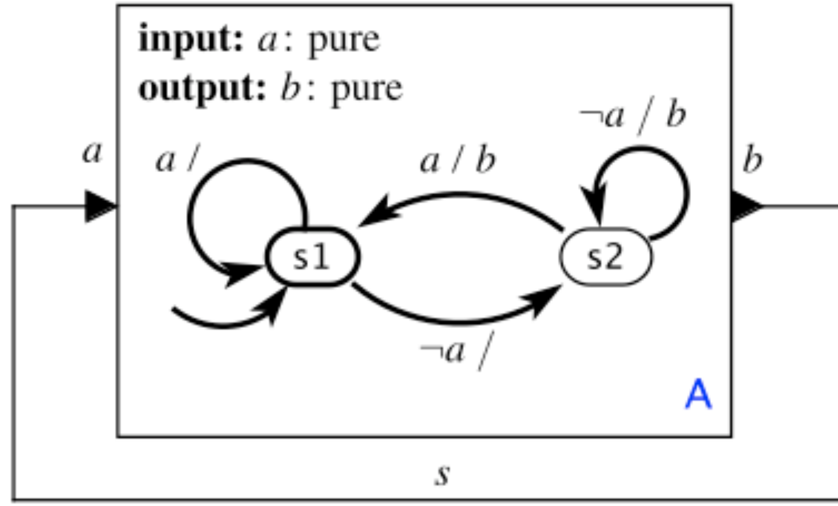


In state **s1**, if the input is unknown, we cannot immediately tell what the output will be. We have to try all the possible values for the input to determine that in fact $s(n) = absent$ for all n.

For non-constructive machines, we are forced to do **exhaustive search**. This is only possible if the data types are finite, and is only practical if the data types are small.

Note: This assumes that our constructiveness analysis does not distinguish between „present with value 0" and „present with value 1". If we would make this distinction, which eg would correspond to a one-hot encoding of the signals, we could statically determine (with partial evaluation) that in s1, a = 0 can never be the case, no matter what transition is taken.

Must / May Analysis

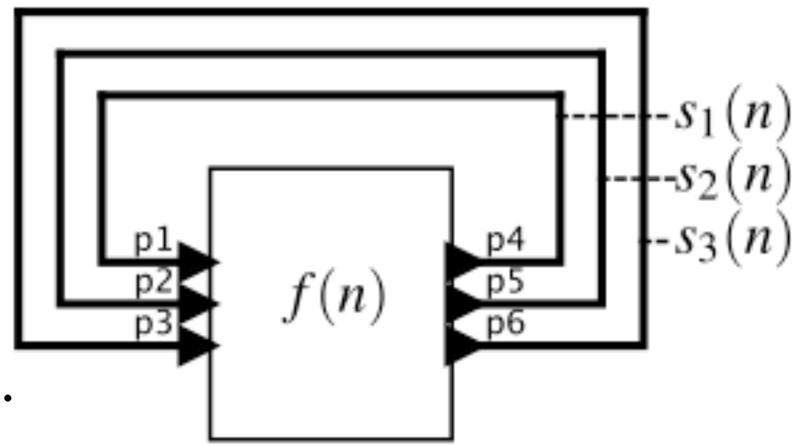


For the above constructive machine, in state s1, we can immediately determine that the machine *may not* produce an output. Therefore, we can immediately conclude that the output is *absent*, even though the input is unknown.

In state s2, we can immediately determine that the machine *must* produce an output, so we can immediately conclude that the output is *present*.

Note: In logical terms, the reasoning for s2 is based on the „law of excluded middle“ (a or ¬a = true), which constructive logic does not include. Similarly, when considering a hardware circuit, this circuit would be not constructive in that it would be delay sensitive. Therefore, Berry (1999) considers the reasoning for s2 „speculative“ and rejects it. However, as Schneider et al. (SLAP 2005) argue, we may be less conservative in software code generation (effectively performing a static partial evaluation), which would consider this machine as constructive.

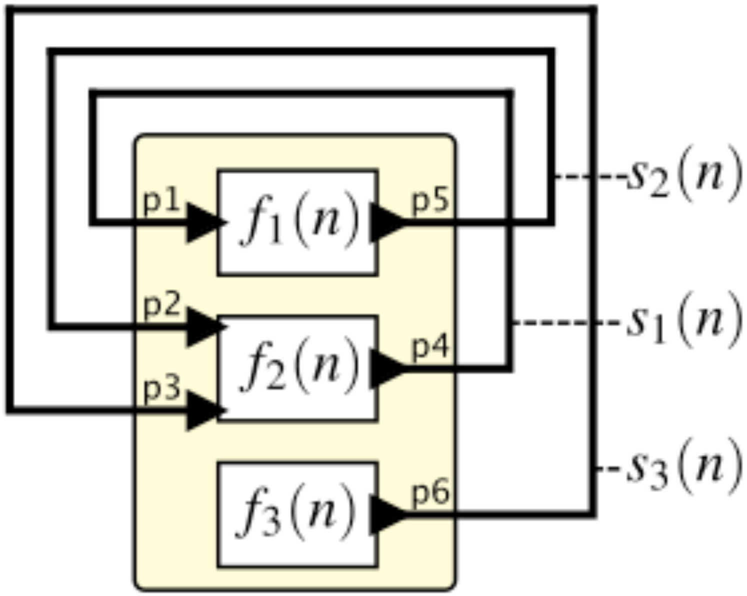
Constructive Semantics: Multiple Signals



1. Start with $s_1(n), \cdots, s_N(n)$ *unknown*.
2. Determine as much as you can about $(f(n))(s_1(n), \cdots, s_N(n))$.
3. Using new information about $s_1(n), \cdots, s_N(n)$, repeat step (2) until no information is obtained.
4. If $s_1(n), \cdots, s_N(n)$ all become known, then we have a unique fixed point and a constructive machine.

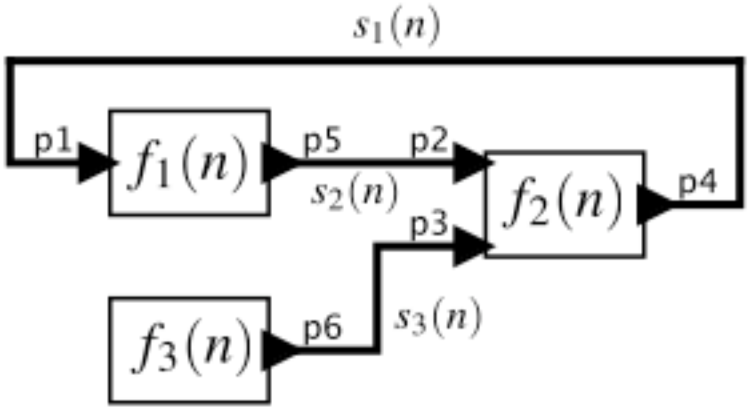
A state machine for which this procedure works is said to be **constructive**.

Constructive Semantics: Multiple Actors



Procedure is the same.

Constructive Semantics: Arbitrary Structure



Procedure is the same.

A state machine language with constructive semantics will reject all compositions that in any iteration fail to make all signals known.

Such a language rejects some well-formed compositions.

Summary

- In a *synchronous composition*, reactions are simultaneous and instantaneous – even if there are causal relationships.
- All actor compositions (side-by-side, cascade) can be regarded as feedback composition.
- We require compatible data types.
- *Well-formed* system has unique *fixed point*, which defines the semantics (behavior) of the system.
- Can break this down into *firing functions*.
- *Constructive* systems allow iterative, constructive procedure to find fixed-point (do not require exhaustive search)

Conclusion

The emphasis of synchronous composition, in contrast with threads, is on *determinate* and *analyzable* concurrency.

Although there are subtleties with synchronous programs, all constructive synchronous programs have a unique and well-defined meaning.

Automated tools can systematically explore *all* possible behaviors. This is not possible in general with threads.